Buckling of Viscoelastic Plates

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Theme

THE purpose of the present study is to develop a simple and sufficiently accurate method which deals with the analysis of buckling of viscoelastic plates of arbitrary geometry. The method developed by Mazumdar[1,2] has earlier been used by the authors[3,4] for a wide class of problems of viscoelastic materials. In the present paper, the method is used to solve the problem of buckling of viscoelastic plates for a large class of in-plane forces and boundary conditions, and in particular the cases of a clamped rectangular plate and a clamped semicircular plate are examined.

Contents

Consider a plate of viscoelastic material with the usual simplifying linearizing assumptions. The plate is acted on by a time-dependent load $q(x,y,\tau)$ in the downward z direction and by in-plane forces N_x, N_y, N_{xy}, functions of x,y, and τ, in the oxy plane. At a particular instant of time τ, the intersections between the deflected surface of the plate and the parallels $z=$constant yield contours which, after projection onto the oxy plane, yield a family of nonintersecting closed curves, known as contour lines of equal deflection.

Consider a portion of the plate bounded by a contour curve $u(x,y)=$constant at any instant of time τ. Summing the tractions around the boundary and the forces on the area Ω_u inside the boundary, the following static equation is obtained.

$$D(P)\frac{\partial^2 V}{\partial u^2}\oint_{C_u} R_I \mathrm{d}s + D(p)\frac{\partial V}{\partial u}\oint_{C_u} F_I \mathrm{d}s + D(p)V$$

$$\oint_{C_u} G_I \mathrm{d}s + V\oint_{C_u}\frac{K\mathrm{d}s}{\sqrt{t}} = \int\int_{\Omega_u} q\mathrm{d}\Omega - \frac{\mathrm{d}w_0}{\mathrm{d}u}\oint_{C_u}\frac{K\mathrm{d}s}{\sqrt{t}} \tag{1}$$

where

$$V=\frac{\partial w}{\partial u} \tag{2a}$$

$$K=N_x u_x^2 + N_y u_y^2 + 2N_{xy}u_x u_y \tag{2b}$$

$$t=u_x^2+u_y^2 \tag{2c}$$

$$w=w(u,\tau) \tag{2d}$$

Here $D(p)$ is a time operator and R_I, F_I, and G_I are functions of u and its derivatives and are given in the main paper.

Consider an equation of the form

$$\frac{\mathrm{d}^2 V_i(u)}{\mathrm{d}u^2}\oint_{C_u} R_I \mathrm{d}s + \frac{\mathrm{d}V_i}{\mathrm{d}u}\oint_{C_u} F_I \mathrm{d}s + V_i\oint_{C_u} G_I \mathrm{d}s$$

$$-\lambda_i^2 V_i\oint_{C_u}\frac{K^*(x,y)\mathrm{d}s}{\sqrt{t}} = 0 \tag{3}$$

Received Dec. 31, 1975; synoptic received Sept. 22, 1976; revision received Dec. 1, 1976. Full paper available from National Technical Information Service, Springfield, Va., 22151, as N77-11443 at the standard price (available upon request).

Index category: Structural Stability Analysis.

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where

$$V_i(u)=\frac{\mathrm{d}w_i}{\mathrm{d}u} \tag{4}$$

This is a Sturm-Liouville equation and possesses a complete set of orthogonal solutions V_i, each corresponding to a value of λ_i, $i=1,2,3,\ldots$, and with the orthogonality relation as

$$\int_{u^*}^{0} V_i V_j \left(\oint \frac{K^*\mathrm{d}s}{\sqrt{t}}\right)\mathrm{d}u = \delta_{ij} \tag{5}$$

Thus $V(u,\tau)$ can be expanded in an infinite set of functions $V_i(u)$ as

$$V(u,\tau)=\sum_{i=1}^{\infty} g_i(\tau)V_i(u) \tag{6}$$

Use of this expansion together with the orthogonality relation Eq. (5) in Eq. (1) will produce an infinite set of coupled equations in $g_i(\tau)$. These equations will readily uncouple when the function $K(x,y,\tau)$ is separable in space and time coordinates as

$$K(x,y,\tau)=N(\tau)\ K^*(x,y) \tag{7}$$

The problem does not become trivial with this simplification; it includes some of the most important classes of problems, in particular, the case of purely time-dependent in-plane forces.

The time equation now reduces to the infinite number of ordinary differential equations

$$\lambda_j^2 D_I(p)g_j(\tau) + D_2(p)\{N(\tau)g_j(\tau)\}$$

$$= D_2(p)\{q_j(\tau) - A_j N(\tau)\} \tag{8}$$

where $q_j(\tau)$ and A_j are the orthogonal series expansion coefficients of $q(u,\tau)$ and $(\mathrm{d}w_0/\mathrm{d}u)$, and D_I and D_2 are linear differential time operators.

The particular problems considered in the main paper are clamped rectangular and semicircular plates of standard linear solid material, with constant loading and in-plane forces, using the form for the lines of equal deflection in accordance with Ref. 5.

To determine the time response of the system, the parameters λ_i must be obtained from Eq. (3). In practice, since the contour integrals occurring in Eq. (3) cannot be evaluated analytically, a numerical evaluation is necessary.

Table 1 Rectangular plate

	Aspect ratio $\phi=b/a$, $m=6$		
ϕ	$\lambda_1^2 a^2$	$\lambda_2^2 a^2$	$\lambda_3^2 a^2$
0.5	18.14	51.55	127.4
0.9	13.38	43.03	94.1
1.0	13.19	42.96	93.4
1.1	13.16	42.91	92.9
1.25	13.15	42.91	92.8
1.5	13.88	44.41	97.4
2.0	15.44	46.93	108.2
3.0	21.08	59.59	–
5.0	40.76	–	–

Table 2 Semicircular plate

m	$\lambda_1^2 a^2$	$\lambda_2^2 a^2$	$\lambda_3^2 a^2$
2	47.0	176.6	–
3	44.9	184.3	345
4	44.9	152.6	–
5	44.9	151.0	330
6	45.2	150.7	328
7	45.2	150.4	312
8	45.2	150.8	312

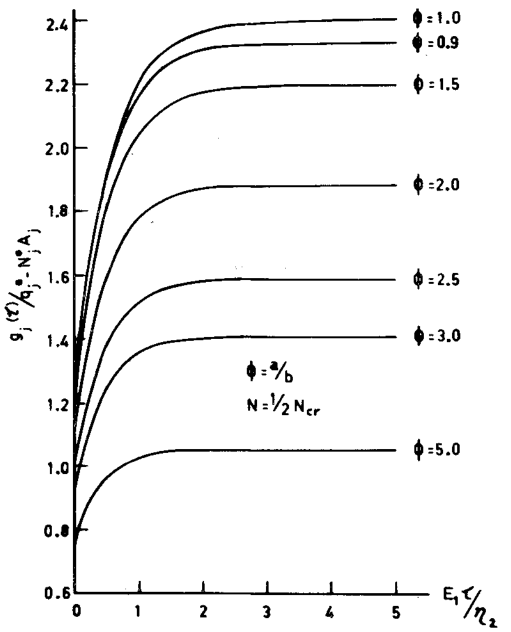

Fig. 1 The behavior of a rectangular plate for varying aspect ratio.

The method of collocation was used to determine the critical buckling loads for an elastic plate

$$\frac{Na^2}{D} = \lambda_j^2 a^2 \tag{9}$$

In Table 1, these parameters are given for various values of the aspect ratio $\phi = b/a$ for the first three modes of the rectangular plate, and for the semicircular plate in Table 2.

The existence of two types of critical load for viscoelastic plates under the action of in-plane forces was shown by Deleeuw.[6] There is a critical buckling load N_{cr} so that instantaneous buckling occurs for $N \geq N_{cr}$, and a semicritical load N_{sc} so that for $N_{sc} \leq N < N_{cr}$ instability occurs, the deflection of the plate increasing with no upper bound as time increases.

The material considered is one which behaves elastically under hydrostatic stress and has the properties of a standard linear solid in shear. In the numerical calculations the material constants are assigned the values

$$E_2/E_1 = 2.6, \quad 3K/E_1 = 5 \tag{10}$$

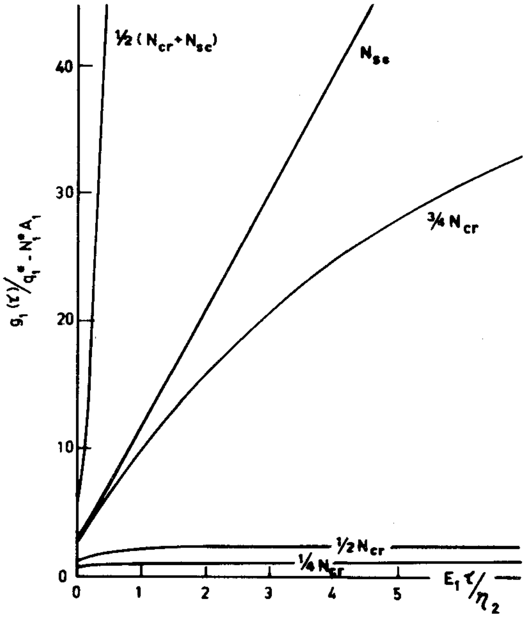

Fig. 2 The time behavior of a square plate under constant inplane force.

The critical loads for the ith modes are then given by

$$N_{sc} = -0.10014\lambda_i^2 E_1 h^3 \tag{11a}$$

$$N_{cr} = -0.13095\lambda_i^2 E_1 h^3 \tag{11b}$$

In Fig. 1, behavior of rectangular plates for a range of values of $\phi = b/a$ is shown for $\frac{1}{2}N_{cr}$. The time behavior of a square plate under a range of constant in-plane forces is shown in Fig. 2.

The time behavior of the semicircular plate will in essence be the same as that of a square plate, with the use of the appropriate value of λ_i^2.

Acknowledgment

This work is based upon a portion of the Ph.D. dissertation submitted by J. S. Hewitt to the University of Adelaide.

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